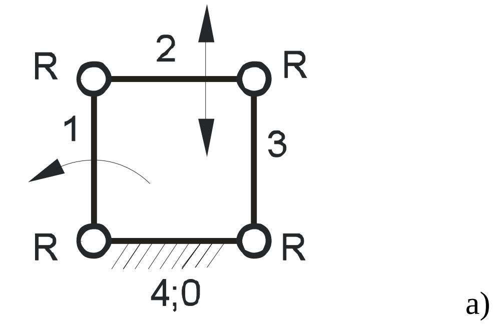
Institut für Getriebetechnik und Maschinendynamik
der RWTH Aachen

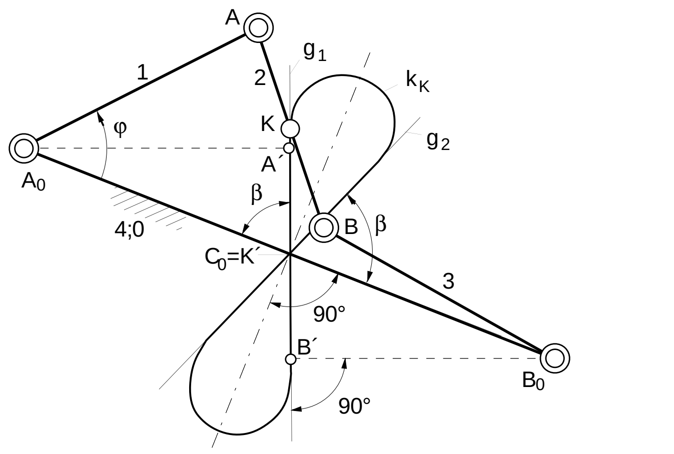
IGM - Mechanism Collection

Watt's Linkage, Evans' Linkage 610

- Guidance Mechanism for transformation of a rotation into an approximate straight line
- Swinging Planar Double-Rocker Mechanism



a)



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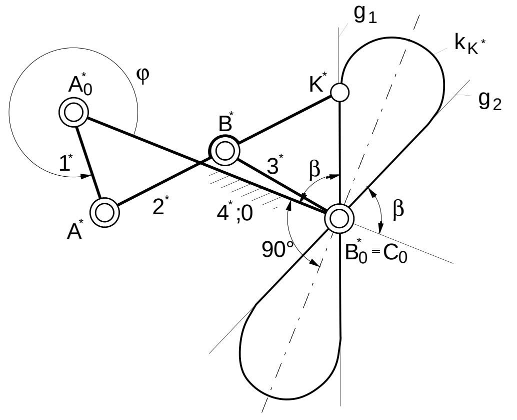

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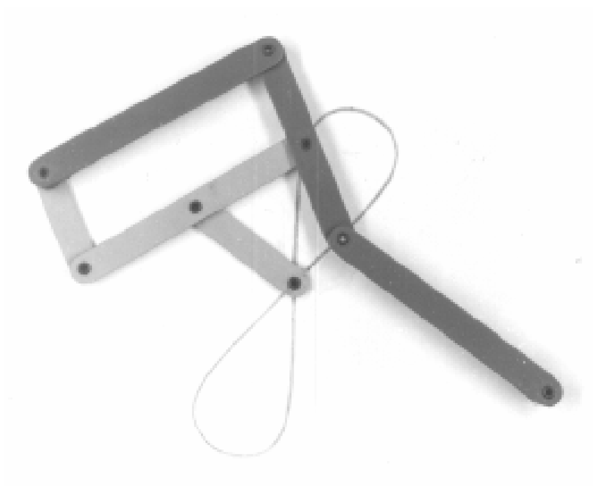

d)

Fig. 1. Watt's-Linkage, Evans'-Linkage

a) Kinematic Chain
b) Kin. Schematic Diagram of Watt's-Linkage
c) Kin. Schematic Diagram of Evans'-Linkage
d) Mechanism Model

Symbols in Kinematic Chain:
R for Rotation **P** for Prismatic **S** for Screw Motion ↶ Input link; ↔ Output link
Example **R_2P**: Joint with 3 degrees of freedom; 2 Rotations, 1 Prismatic

Characteristics:

Number of Input Links	: 1, of which 1 at Frame
Number of Output Links	: 1, of which 0 at Frame
Number of Links	: 4, of which 4 binary
Number of Joints	: 4, of which 4 Hinges (R)

Dimensions (in Unit Lengths):

Watt's Linkage

$\overline{A_0A} = l_1 = 5$; $\overline{AB} = l_2 = 4$; $\overline{B_0B} = l_3 = 5$;

$\overline{A_0B_0} = l_4 = 10{,}77$; $\overline{AK} = k = 2$; $\overline{BK} = l = 2$.

Evans' Linkage

$\overline{A_0^*A^*} = l_1^* = 2$; $\overline{A^*B^*} = l_2^* = 2{,}5$; $\overline{B_0^*B^*} = l_3^* = 2{,}5$;

$\overline{A_0^*B_0^*} = l_4^* = 5{,}39$; $\overline{A^*K^*} = k^* = 5$; $\overline{B^*K^*} = l^* = 2{,}5$.

Description:
Since the beginning of the development of the steam engine, Watt's Linkage and Evans' Linkage are well known for their path and approximate straight guidance (**fig. 1**). The model in fig. 1d shows that both Watt's Linkage (dark linkage) and Evans' Linkage (bright linkage), which are according to Roberts constructed as cognate linkages, produce the same figure-eight-shaped coupler-point curve. To prove this by demonstration, both linkages are combined by use of a revolute pair that acts as the path following coupler point.

Author: Prof. Dr.-Ing. G. Dittrich

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The Watt-Linkage (fig. 1b) is a symmetric, swinging double-rocker mechanism A_0ABB_0 with $\overline{A_0A} = \overline{B_0B}$. The coupler point K producing the figure-eight-shaped coupler curve is in the middle of coupler $\overline{AB}$ ($\overline{AK} = \overline{BK}$). The dashed lines show the initial position $A_0A'B'B_0$ for which the coupler $A'B'$ is perpendicular to the rockers A_0A' and B_0B'. The coupler point K′ then intersects with the middle C_0 of the fixed link $\overline{A_0B_0}$ ($\overline{A_0C_0} = \overline{B_0C_0}$). With the rocker lengths $\overline{A_0A} = l_1$, $\overline{B_0B} = l_3$, the coupler length $\overline{AB} = l_2$ and considering $l_1 = l_3$ and $\overline{AK} = \overline{BK} = l_2/2$ the length of the fixed link becomes

$$\overline{A_0B_0} \equiv l_4 = \sqrt{4l_1^2 + l_2^2} \quad . \qquad (1)$$

The figure-eight-shaped coupler curve k_K of the coupler point K is symmetric in C_0 with respect to the fixed link A_0B_0 and to the line perpendicular to A_0B_0 in C_0; in C_0 there is a double point. Both lines g_1 and g_2, which seem to intersect the coupler curve for a great part (approximated straight guide of coupler point K) are fifth order tangents in C_0 and have an angle β bzw. (180° - β) with respect to the fixed link A_0B_0, where the angle β can be calculated by

$$\tan\beta = (2\, l_1) / l_2 \, . \qquad (2)$$

The line g_1 intersects with the coupler line $A'B'$ when the mechanism is in the mentioned initial position.

The in fig.1c presented Evans-Linkage $A_0{*}A{*}B{*}B_0{*}$ is one of the two cognate mechanisms which according the Roberts Theorem exist. Hereby the joints $A_0{*}$ and A_0 intersect just as the joints $B_0{*}$ and C_0. The Evans-Linkage is an isosceles, swinging double-rocker mechanism with $\overline{B_0{*}B{*}} = \overline{A{*}B{*}}$. The coupler point K* that produces the same figure-eight-shaped coupler curve ($k_{K*} = k_K$) is on the coupler line A*B* at a distance of $\overline{B{*}K{*}} = \overline{A{*}B{*}}$ from B*. The kinematical dimensions of Evans' Linkages can be obtained from the cognate Watt's Linkage and turn out to be

$$\overline{A_0^*A^*} \equiv l_1^* = 0{,}5\, l_2 \; , \qquad \overline{A^*B^*} \equiv l_2^* = 0{,}5\, l_1 \; ,$$

$$\overline{B_0^*B^*} \equiv l_3^* = 0{,}5\, l_1 \; , \qquad \overline{A_0^*B_0^*} \equiv l_4^* = 0{,}5\, l_4 \; ,$$

$$\overline{A^*K^*} \equiv k^* = l_1 \; , \qquad \overline{B^*K^*} \equiv l^* = 0{,}5\, l_1 \; .$$

Therefore the Evans-Linkage can be constructed within a smaller space compared to the cognate Watt-Linkage.

Calculation of the Coupler Curve

The calculation of the coupler curve k_K of the Watt-Linkage depends on the input angle $\varphi = \sphericalangle B_0A_0A$ between the boundaries

$$\varphi = \pm \arccos \left[(l_1^2 + l_4^2 - (l_2 + l_3)^2) / (2 l_1 l_4) \right]$$

which can be obtained from eq.(1). Since $l_1 = l_3$ and $\overline{AK} = \overline{BK} = l_2/2$ the co-ordinates of the coupler curve follow from (**fig. 2**):

$$\overline{B_0A} = f = \sqrt{l_1^2 + l_4^2 - 2 l_1 l_4 \cos\varphi} \;\; ;$$

$$\cos\overline{\psi_s} = \left(l_1^2 + f^2 - l_4^2\right) / (2 l_1 f) \; ,$$

$$\sin\overline{\psi_s} = (l_4 \sin\varphi) / f \;\; ;$$

$$\cos\overline{\psi_t} = \left(l_2^2 + f^2 - l_3^2\right) / (2 l_2 f) \;\; ;$$

$$\sigma = \varphi + \overline{\psi_s} + \overline{\psi_t} - \pi \quad \text{(coupler angle)} \;\; ;$$

$$x_K(\varphi) = l_1 \cos\varphi + \frac{l_2}{2}\cos\sigma \; , \quad y_K(\varphi) = l_1 \sin\varphi + \frac{l_2}{2}\sin\sigma \; .$$

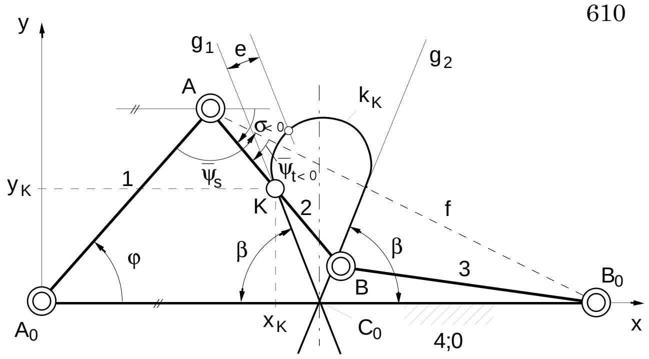

Fig. 2. Calculation of the coupler curves

The angle $\overline{\psi}_s$ can be uniquely determined by its sine and cosine values, however care must be taken by the calculations of angle $\overline{\psi}_t$ since the sign of the angle depends on the position of coupler 2 with respect to the diagonal B_0A.

By use of relation (2) the equation of the line g_1 (and g_2) become

$$y = \overset{-}{\scriptstyle(+)} \frac{2l_1}{l_2} x \overset{+}{\scriptstyle(-)} \frac{l_1 l_4}{l_2} \quad ,$$

The error e of the coupler curve with respect to this line, which also defines the accuracy of the straight line guidance around the double point C_0, dependent of the input angle φ or the distance

$$\overline{C_0K} = s = \sqrt{\left(\frac{l_4}{2} - x_K\right)^2 + y_K^2}$$

can be found by (**fig. 3**):

$$e = \left| \frac{2l_1}{l_4} x_K \overset{+}{\scriptstyle(-)} \frac{l_2}{l_4} y_K - l_1 \right| \quad .$$

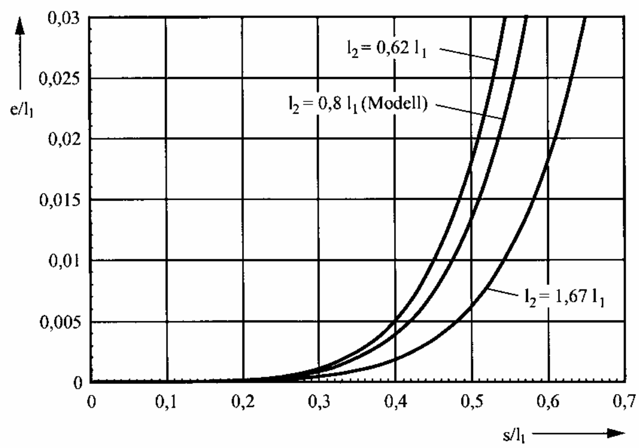

Fig. 3. Error e of the coupler curve k_K with respect to the line g_1 depending on distance $s = \overline{C_0K}$.

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